MTH501 Linear Algebra

Final Term Examination - February 2005
Time Allowed: 150 Minutes

Instructions

Please read the following instructions carefully before attempting any of the questions:

1. Attempt all questions. Marks are written adjacent to each question.
2. Do not ask any questions about the contents of this examination from anyone.
   a. If you think that there is something wrong with any of the questions, attempt it to the best of your understanding.
   b. If you believe that some essential piece of information is missing, make an appropriate assumption and use it to solve the problem.

****WARNING: Please note that Virtual University takes serious note of unfair means. Anyone found involved in cheating will get an `F` grade in this course.**

Total Marks: 65 **Total Questions: 12**

Question No. 1	**Marks : 2**

If {*v1, v2, v3,…,vn*}be the orthogonal set of vectors then which statement(s) must be true.

I. vi . vj = 0 for all.

II. vi . vi >0 for all *i and* $v_i \neq 0$.

III. Set{ *c1v1,c2 v2,c3 v3,…,cnvn*}

1 I only.
2 I and II only.
3 II and III only
4 All of the three.

Question No. 2	**Marks : 10**

Find a least squares solution of the inconsistent system Ax =b where

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}.$$

Question No. 3	**Marks : 5**

Find orthogonal projection of

$$y = \begin{bmatrix} 7 \\ 6 \end{bmatrix} \text{ Onto } u = \begin{bmatrix} 4 \\ 2 \end{bmatrix}.$$

Question No. 4	**Marks : 2**

If matrix A has zero as an eigenvalue then which statement(s) about A must be true.

I. Matrix A is not invertible.
II. Matrix A will also have an eigenvalue 2.
III. Matrix is diagonalizable.

1 II and III only.
2 I only.
3 II and III only.
4 All three.

Question No. 5	**Marks : 2**

A Linear transformation $T:R^2 \to R^2$ is such that

$$T[e_1] = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \;\&\; T[e_2] = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \text{ then } T\begin{bmatrix} 1 \\ -3 \end{bmatrix} \text{ is,}$$

1 $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

2 $\begin{bmatrix} 1 \\ -3 \end{bmatrix}$

3 $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$

4 $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$

Question No. 6	**Marks : 10**

Determine whether the subset

$$\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix}?$$

If so find the corresponding eigenvector.

Question No. 11	**Marks : 10**

Show that the signal 2^k ***and*** $(-4)^k$ are the solution of the difference equation
$y_{k+2}+2y_{k+1}-8y_k = 0$.

Find a basis {***u1***, ***u2***, ***u3***} for R^3 such that ***P*** is the change-of-coordinates matrix from {***u1***, ***u2***, ***u3***} to the basis {***v1***, ***v2***, ***v3***}.

Question No. 12	**Marks**	**: 5**

Find the vector ***x*** such that we have

$$[x]_B = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

(Coordinate matrix of ***x*** with the basis B), where

$$B = \left\{ \begin{bmatrix} 3 \\ -5 \end{bmatrix}, \begin{bmatrix} -4 \\ 6 \end{bmatrix} \right\}.$$